Example application for rational approximations

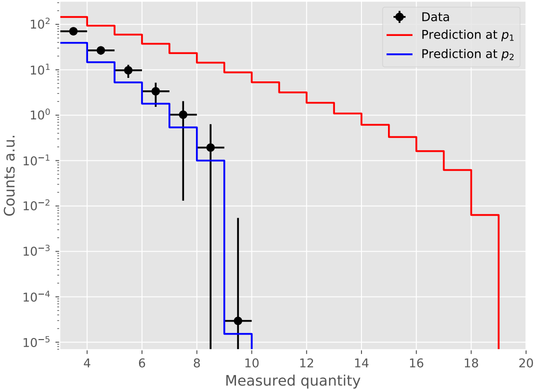
Mohan Krishnamoorthy, Sven Leyffer, **Holger Schulz**

Scidac weekly meeting, Fermilab, March 21, 2019

Motivation

Typical problem in HEP: parameter scan of a physics model in multiple dimensions and comparison with data.

- data is in histogram, physics prediction also as histogram
- different predictions for different physics parameters, p
- What points in the parameter space give predictions that are in statistical agreement with data?



Likelihood

A good measure to test compatibility with data and prediction is a likelihood such as

$$\mathcal{L}(p) = \prod_b \frac{N_b(p) \cdot \lambda_b \cdot e^{N_b(p)}}{\lambda_b!}$$

where

- b runs over all bins of the histogram
- λ_b denotes the value of bin b in the data histogram
- $N_b(p)$ denotes the value of bin b in the physics prediction histogram, evaluated at the physics point p

We now need to numerically find the point $\hat{p}$ which maximises the likelihood function. That would be the best fit point.

Likelihood-scan

Usually, we are not just interested in the point $\hat{p}$ but in confidence regions, especially in the case of degeneracies

- typically 1σ and 2σ contours
- Those are regions in the parameter space that are also statistically compatible with the measured data

Scanning the parameter space can get expensive as the predictions $N_b(p)$ can come from arbitrarily complex simulations.

- In the following: 3D parameter scan performed with MultiNest in MPI mode.

Our example

We simulate dark matter signals in a Xenon detector. There are 3 parameters:

- m_χ the mass of the dark matter candidate
- c_π and c_+ are coupling strengths

For the approximations:

- we evaluate the simulation at 500 randomly sampled points, P, yielding 500 sets of $N_b(p)$
- we fit separate approximations $a_b(p)$ for each bin b using the $N_b(p) \, \forall \, p \in P$

Comparison

Since the likelihood is a 3D function, visualisation is best done as projections onto 1 or 2 dimension. We use a measure called profile-likelihood to do that. The point of maximum likelihood will be displayed as a star.

In the following plots, we compare results obtained using the true simulation with those obtained using different approximations.

Reminder: this is for the true simulation

$$\mathcal{L}(p) = \prod_b \frac{N_b(p) \cdot \lambda_b \cdot e^{N_b(p)}}{\lambda_b!}$$

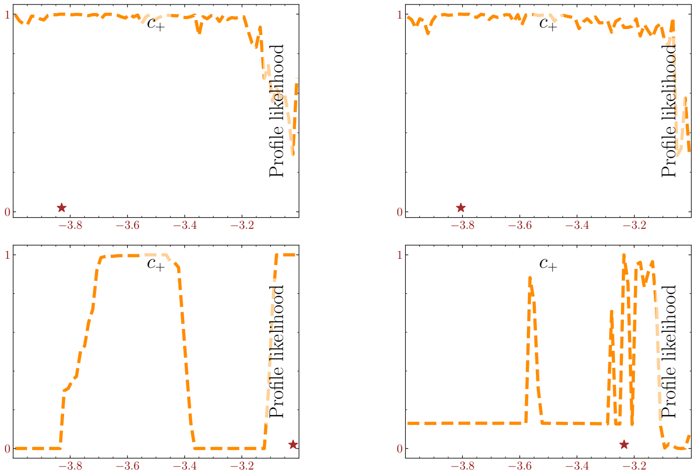
This is the likelihood when using approximations $a_b(p)$

$$\mathcal{L}(p) = \prod_b \frac{a_b(p) \cdot \lambda_b \cdot e^{a_b(p)}}{\lambda_b!}$$

There are always three approximations in the following plots, a pole-free rational approximation, a polynomial approximation and a rational approximation with poles.

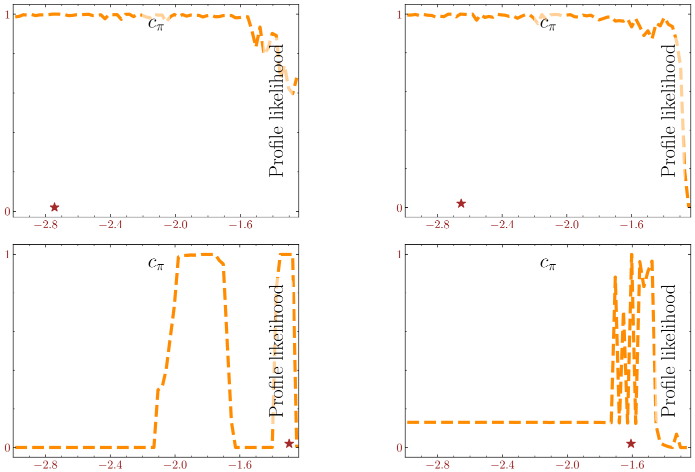
1D Profile likelihoods

true simulation	pole-free rational
polynomial	rational with poles



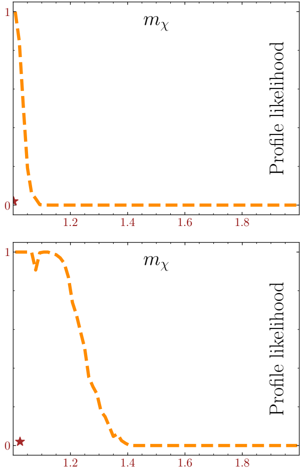
1D Profile likelihoods

true simulation	pole-free rational
polynomial	rational with poles



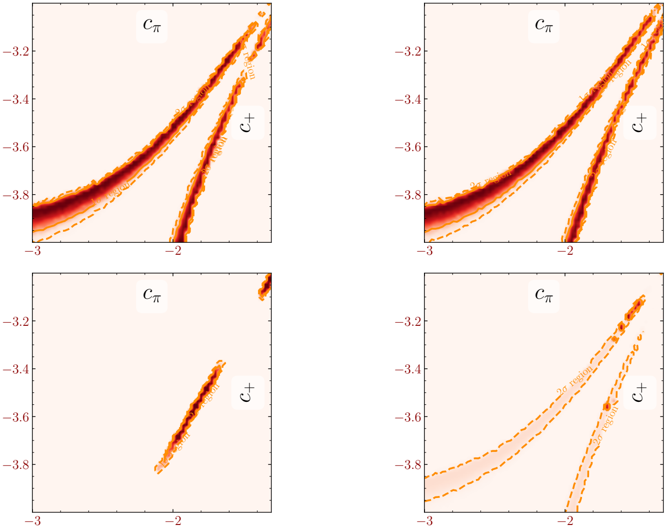
1D Profile likelihoods

true simulation	pole-free rational
polynomial	rational with poles



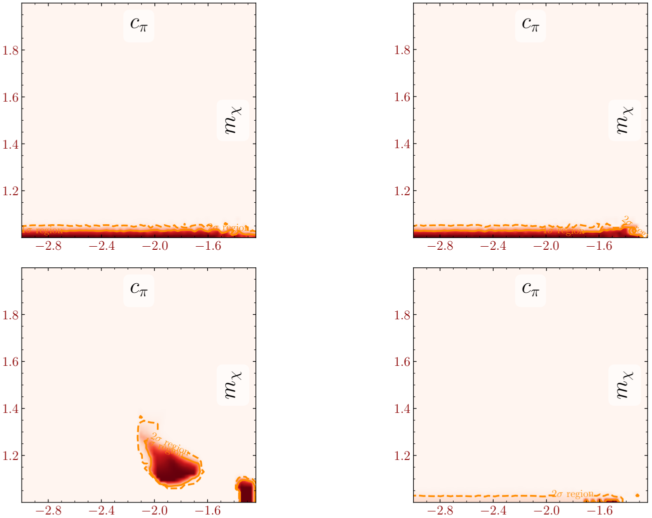
2D Profile likelihoods

true simulation	pole-free rational
polynomial	rational with poles



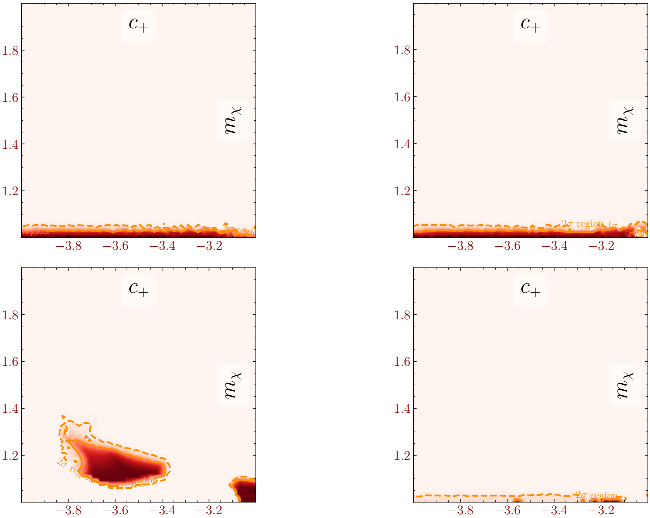
2D Profile likelihoods

true simulation	pole-free rational
polynomial	rational with poles



2D Profile likelihoods

true simulation	pole-free rational
polynomial	rational with poles



Remarks

The likelihood scan takes about an hour for the true simulation on my laptop (the simulation is quite simple in this case).

With approximations, it takes less than 2 minutes.

The number of likelihood evaluations in each scan is approximately the same (20k).

We find that the pole-free rational approximations do a very good job.

Rational approximations with poles, unsurprisingly, are a less than optimal choice.

We see the breakdown of the polynomial approximation quite vividly.